m_2 is the atomic weight of the surface atom For a relatively heavy atom such as Au, the energy transfer is poor when the impacting particle is N_2 As a consequence, one would expect the Ag_2S surface to have a higher sputtering yield than Au

References

[1] McKeown, D , Cabezas, A Y , and Adelson, H E , "Low energy sputtering studies," Bull Am Phys Soc **6**, 359 (1961)

[2] McKeown, D , Fox M G and Schmidt, J J Measurement of surface erosion from Discoverer 26 " ARS J **32**, 954–955 (1962)

[3] McKeown, D , ' Surface erosion in space," *Rarefied Gas Dynamics*, edited by J A Laurmann (Academic Press, New York 1963) pp 315–326

[4] *U S Standard Atmosphere* (U S Government Printing Office, Washington, D C , 1962), p 81

Technical Comments

Comment on "Propulsion Application of the Modified Penning Arc Plasma Ejector"

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IN a recent paper,[1] results of a brief analytical and experimental treatment of the modified Penning plasma accelerator are presented which purport to show that devices of this general class are not attractive for propulsion purposes The writers have previously called attention to a number of factors that render the conclusions of Ref 1 invalid§ and wish to criticize again the more significant points These include use of a single-fluid model in the theoretical approach which cannot account for the essential features of the problem, neglect of dominant diffusion processes in the interpretation of observed experimental trends, and use of a cold cathode discharge that can be shown by a priori arguments to be inherently inefficient and cannot be considered for propulsion purposes

Criticism of Theory

In the theoretical treatment of the effect of the exit fringing magnetic field on the divergence of the exhaust beam, it is stated in Ref 1 that "the foregoing calculations are certainly of the simplest type that can be applied in a basically complex plasma situation, and it is reasonable to question their validity " The writers wish to underscore this statement For example, the plasma model employed is that of a uniform, single-component, MHD flow in which transport coefficients are assumed to be scalar quantities In the problem considered, however, if a fluid model is to be employed, it is necessary that the model consist of a two component flow with spatial inhomogeneities allowed, with tensor transport properties, and with coupling between the ion and electron fluids caused only by means of volume electrostatic fields and not by collisions Such a flow is indeed complex and, yet, still does not allow for the effects of a nonthermal velocity distribution; hence, it is generally desirable to adhere to the self consistent, individual-particle approach (e g , Refs 2 and 3) Virtually all of the essential details of the physical processes occurring in the plasma are lost when the single-fluid simplifications employed in Ref 1 are applied

Ion densities of $10^{12}/cm^3$ and electron energies of the order of 10 ev are representative of typical beam conditions for practical propulsion devices of this type Using the calculations of Ref 1, one then arrives at electron mean free paths (μ) on the order of 30 cm, a distance large compared to characteristic apparatus dimensions Both of these lengths are large compared to the electron cyclotron radius (~0 03 cm) Hence, for conditions of interest for propulsion, electrons having an appreciable transverse kinetic energy are trapped on field lines (in a classical sense) and can migrate across field lines as a result of $\mathbf{E} \times \mathbf{B}$ drifts (Hall currents) or diffusion (either classical or anomolous) If one adopts the fluid model, this implies that conductivity is a tensor property of the medium and cannot be considered simply (as in Ref 1) the reciprocal of the resistivity The conductivity transverse to a magnetic field is reduced by a factor $(\nu/\omega)^2$ for $\nu/\omega \ll 1$ from the values used in Ref 1 The reduction by a factor of 3 in conductivity (taken in Ref 1 from Spitzer) to account for the magnetic field is appropriate only when applied to resistivity and only under conditions of $\nu/\omega > 1$ Furthermore, the use of the Spitzer formula for the conductivity along the magnetic field implies that the energy gain between collisions is small, i e , when $E\mu > kT/e$, the theory is not valid In order for the plasma source to operate as a propulsion device, it is necessary that electric fields accelerate ions and that these electric fields exist over the source dimensions; hence, for conditions of interest, $Ee\mu/kT \approx 30$ Clearly the conditions needed for the concept of a scalar conductivity to be meaningful do not exist in the propulsion version of the Penning source where Hall effects are important and where the energy gain between collisions can be greater than the energy of random electron motion

In the theory of Ref 1, it is assumed that currents due to electron motions exert $\mathbf{J} \times \mathbf{B}$ forces on the plasma, causing changes in the momentum of the ions However, it must be understood that, because an essentially collisionless situation exists, the only way in which the motion of the ions can be

Received July 30, 1963

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§ Discussion following presentation of Ref 1 at the ARS Electrical Propulsion Conference, Berkeley, Calif , March 14–16, 1962 affected is by means of a macroscopic electric field, which, by symmetry, must have only radial and axial components The presence of electric fields is completely neglected in Ref 1, as is the resulting $\mathbf{E} \times \mathbf{B}$ azimuthal current drifts that would largely conteract the j_θ included in Ref 1 Such internal electric fields would, of course, occur only in a two-fluid model with large ion cyclotron radius and not in the single-fluid picture

Perhaps the most serious inadequacy of the theory presented in Ref 1 is the assumption that the exhaust beam electrons constitute a spatially homogeneous, monoenergetic drift Although the authors touch on the effects of spatial inhomogeneity and nonthermal velocities, this point should be elaborated since it is essential to correct operation of the source both in space and in the laboratory A high-energy flux of electrons emanates from the cathode surface; portions of this flux can pass through the accelerator without undergoing collisions because of the large mean free path For the most part, electrons that neutralize the external beam originate from this source In most designs the cathode is located on the axis of the accelerator, and the neutralizing current is drawn from a region of small radial magnetic field (Magnetic flux lines close to the axis ultimately diverge but at axial distances at which the electron cyclotron radius is large, and electrons are no longer trapped) Hence, a relatively high-current, high-energy, nonthermal supply of electrons is available for neutralization If, in the laboratory, the plasma source is electrically isolated from the test chamber walls and target, the potential of the cathode (source) will automatically adjust to a value such that the exhaust beam electron current drawn from this emission-limited supply exactly equals the ion current In the case of an isolated vehicle in space, the net charge on the vehicle would adjust to preserve current neutrality Small axial potential drops may exist in the exhaust beam, but these will be of the order of the external electron energy (~10 ev) Because electrons and ions couple primarily through macroscopic electric fields, such drops are not due to plasma resistivity, but to the same mechanism of electrostatic field generation[2 3] which occurs within the plasma source As axial distance from the source increases, such potential gradients will retard electrons (and accelerate ions slightly) to the point where the conditions for current and space-charge neutrality are satisfied by having the electron drift velocity equal the ion beam velocity If tank walls and collectors are located sufficiently remote from the magnetic field such that electron cyclotron radii are large, and if the forementioned isolation precautions are observed, satisfactory beam emergence and thrust generation can be achieved This has been demonstrated repeatedly in the writers' laboratories and reported in the classified literature

Criticism of Experiment

The authors state that the experimental Penning source reported in Ref 1 "apparently would support oscillating electron trajectories in the conventional way " This statement implies that measurements were not made to determine if electrostatic voltage gradients necessary for ion acceleration were maintained in the discharge It has been pointed out by the writers[2 3] very strongly that, if sufficient care is not taken to assure the existence of such gradients, the entire principle of ion acceleration is violated

The most serious criticism, however, concerning the experimental verification of the theory of beam spread in the diverging magnetic field is in the manner in which the results were obtained Apparently, the experiment was conducted by varying the total magnetic field strength, which, in turn, changed the absolute magnitude of the gradient of the diverging region of the magnetic field It has been pointed out in the literature that discharges of this type are quite sensitive to changes in magnetic field strength This is understandable in view of the results of Lehnert[4] and those of Kadomtsev,[5] which show a sudden onset of instabilities at a critical field strength in a magnetically confined plasma When instabilities, such as plasma oscillations or MHD waves, are present in the plasma, the oscillating electrons are quickly thermalized and can rapidly lose their axial momentum This thermalization drastically changes the electrostatic potential gradient in the engine, with consequent changes in the thrust and efficiency The experiments reported in Ref 1 ignore the possibility of an important dependence of engine performance on magnetic field strength Therefore, any data that purport to show the effect of the diverging magnetic field, while ignoring the effect of the level of magnetic field strength on the properties of the plasma or on the discharge characteristics of the engine itself, are certainly subject to question Changes in engine performance were, in fact, noted by the experimenters who state that "although the operating characteristics of the arc vary due to the change in magnetic field intensity, the large change in the low thrust level is considered as supporting evidence of the difficulty of plasma ejection through a divergent magnetic field " Therefore, even though changes in engine performance were noted, the significance of these effects may bear no relation to the assumed cause

Criticism of Use of a Cold Cathode for Propulsion

A propulsion device of the form employed in Ref 1 cannot use a cold cathode without encountering prohibitive penalties in efficiency This can be readily shown by the following simple argument Proper operation of an electrostatic propulsion device requires that $i_{+B} = i_{B}$, where i_{+B} and i_{B} are the exhaust beam ion and electron currents Furthermore, since only the most energetic electrons can leave the discharge and since the highest-energy electrons are those which are emitted by cathode, $i = (1 + K)i_{eB}$, where i is the total electron current emitted by the cathode, and K, $(K > 0)$ represents the fraction of electrons emitted at the cathode which are scattered (elastically or inelastically) and diffuse to the anode The total current to the cathode (and hence in the main power supply circuit) is given by $i = i_e + i_+$, where i_+ is the cathode ion impingement current Now for a cold cathode discharge depending upon secondary emission, $i_e = \gamma i_+$ Available data on secondary electron emission due to argon and alkali metal ions incident on most materials indicate that values of γ below 0 2 are to be expected for the energy ranges of interest From the foregoing equations, $i = (1 + K)(\gamma + 1)\, i_B/\gamma$ Over-all efficiency will be, in general, less than the ratio of beam power P_B to the discharge power $P = iV$ because of beam nonuniformity, and hence

$$\eta \leqslant \frac{P_B}{P} \leqslant \frac{i_{+B}}{i} = \frac{1}{1+K}\left(\frac{\gamma}{1+\gamma}\right)$$

where the second inequality is also a result of beam nonuniformity Hence the use of a cold cathode would imply that, even if all other sources of loss are neglected, the maximum efficiency could not exceed 17% In view of these considerations, the poor results obtained with the plasma accelerator tested in Ref 1 are certainly to be expected It is obvious, therefore, that thermionic cathodes similar to those reported in Refs 6 and 7 must be used if accelerators of this general type are to be considered for propulsion purposes It is this consideration, and not the problem of sputtering, as suggested in Ref 1 which, dictates the choice of cathode type

Several further advantages accrue from the use of a thermionic cathode Voltage current curves for the cold-cathode discharge are shown in Figs 4 and 6 of Ref 1 The data spread over two orders of magnitude in current for a discharge in cesium are apparently indicative of the effect of changes in secondary emission coefficient with surface coverage With argon (Fig 6 of Ref 1), the data lie along a particular characteristic curve However, it is apparent that there is no way of operating the argon cold-cathode discharge at arbitrary combinations of current and voltage The power supply impedance fixes the operating point On the other hand, with a thermionic cathode, a second control parameter is available, i e , the heater power This permits selection of both voltage and current over rather wide ranges and allows the discharge parameters to be adjusted to match the desired mass flow and specific impulse

References

[1] Kilpatrick, W D Mullins, J H , and Teem, J M , ' Propulsive applications of the modified Penning arc plasma ejector,' AIAA J **1**, 806–813 (1963)

[2] Salz, F , Meyerand R G , Lary, E C , and Walch, A P , Electrostatic potential gradients in a Penning discharge " Phys Rev Letters **6**, 523–525 (1961)

[3] Meyerand, R G , Salz, F , Lary, E C , and Walch, A P , ' Electrostatic potential gradients in a non thermal plasma," *Proceedings of the International Conference on Ionization Phenomena in Gases* (North-Holland Publishing Co , Amsterdam, 1962), Vol II

[4] Lehnert, B , Diffusion processes in the positive column in a longitudinal magnetic field," Proc 2nd United Nations Conf Peaceful Use of Atomic Energy **32**, 349–357 (1958)

[5] Kadomtsev, B B and Nedospasov, A V , ' Instability of the positive column in a magnetic field and the 'anomalous' diffusion effect," J Nucl Energy **1**, 230–235 (1960)

[6] Davis, J W , Walch, A P , Meyerand, R G , Salz, F and Lary, E C , Theoretical and experimental description of the oscillating electron ion engine, ' IAS ARS Preprint 61 103-1797 (June 1961)

[7] Davis J W , Angelbeck A W , and Pinsley, E A , Research on the oscillating-electron plasma source," Aeronaut Res Labs Rept ARL 62-471, Wright-Patterson Air Force Base, Ohio (December 1962)

Comment on "Matrices for the Direct Stiffness Method"

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REFERENCE 1 may well be a vital contribution leading toward the second generation of stiffness methods for structural analysis An academic attitude seems appropriate, because obviously no method is trustworthy unless the solution is accurate using small elements Melosh presents new criteria governing the shape functions (displacement functions of Ref 1) to ensure this Perhaps he could comment on an alternative to his subdivision criterion Suppose a shape function gives zero shear stress near a node and appreciable shear stress elsewhere Such shape functions may look very smooth But as the element size is reduced, the stress continues to oscillate violently instead of converging to a continuous function Conversely, if the shape functions are chosen so that they can describe any necessary state of constant stress or curvature (for small elements at least), then the true physical situation is faithfully represented in the limit This criterion is included in that of Melosh It does not necessarily insure monotonic convergence, except perhaps near the limit, whereas the Melosh criterion will ensure monotonic convergence even for large elements

Another reason why a solution may converge to the wrong values is that rigid body motions are not represented Then, as Melosh observes, equilibrium conditions are falsified This elementary observation had not been published previously, to the authors' knowledge, let alone received the attention it deserves; yet in practice it is the most difficult criterion to satisfy Shape functions are invented rather than derived, and despite the criteria there is no unique choice The authors submitted a note to the *AIAA Journal* some months ago, discussing shape functions for a thin curved beam and for a flat quadrilateral plate in bending The difficulties suggested that further progress would be limited without an alternative and simpler technique

A straightforward method of ensuring equilibrium is proposed Assume the deflections are

$$\sum_{i=1}^{N} \alpha_i \ \varphi_i(x,y,z) = [\varphi']\{\alpha\}$$

Here the φ_i are vector functions defined over the element They are generalized shape functions, chosen without regard for the nodal deflections The α_i are scalar multipliers known as generalized deflections They are related to the nodal deflections x_j; thus

$$\{x\} = [T]\{\alpha\}$$

where $[T]$ is easily calculated as an $n \times N$ rectangular matrix, where $N \geqslant n$ Using either orthodox methods or numerical integration, the strain energy may be expressed in terms of a generalized stiffness matrix $[S]$:

$$\tfrac{1}{2}\alpha'[S]\alpha$$

The information latent in $[T]$ and $[S]$ is interpreted as follows:

$$\begin{bmatrix} -S & T' \\ T & 0 \end{bmatrix}^{-1} = [Z]^{-1}$$
$$= \begin{bmatrix} G & t' \\ t & K \end{bmatrix}$$

$[K]$ is the required nodal stiffness No use for $[G]$ has been found, but $[t]$ is needed in deriving stresses, etc ; $\{\alpha\} = [t']\{x\}$

The following conditions are sufficient for nonsingular $[Z]$, and are independent of whether $[K]$ is singular

1) It may be possible to select many subsets of n generalized shape functions which give independent nodal deflections At least one subset must satisfy further conditions

2) The remaining $(N - n)$ shape functions give $(N - n)$ different systems of stresses These must be mutually independent; that is, they cannot be combined to give zero total strain energy

3) The n stress systems under 1 must not combine with any individual stress system under 2 to give zero total strain energy

Thus, shape functions under 1 which are pure translations and rotations are not forbidden, so long as they are independent The proposal is that the first few shape functions should represent the necessary rigid body motions The remainder should then represent the types of deformation envisaged, preferably with minimum rigid-body displacement

Another question which interests the authors concerns natural frequencies Just as the strain energy is expressed in terms of the stiffness matrix, so the kinetic energy may be expressed in terms of a nondiagonal mass matrix, based on the shape functions These two matrices can be combined in an eigenvalue calculation The question is, do the resulting frequency estimates converge monotonically as the mesh is subdivided? One would expect this, because the Melosh criterion means that simple constraints are progressively relaxed In a similar way, work done against second-order strains may be written in matrix form The eigenvalue calculation then gives the buckling load Since only the largest eigenvalue is significant, this case seems clearer

Received September 11, 1963; revision received October 18, 1963

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